Newtonian Mechanics of Economic Growth and Crisis

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Abstract: In 2008, financial services firm Lehman Brothers faced an unprecedented loss due to the continuing subprime mortgage crisis. The firm filed for Chapter 11 bankruptcy protection on 15 September 2008. It was the largest bankruptcy in US history holding over US$600 billion in assets.

Financial services firm Lehman's bankruptcy in late 2008 followed losses due to the subprime mortgage crisis. It was expected to cause just some depreciation in the price of commercial real estate, but in effect it sparked a chain-reaction on financial institutions worldwide. Shortly after we got a global financial crisis, which could be glossed over only with gigantic governmental cash infusions, but it is still going on.

Mathematical theory in macroeconomics since got under some doubt. As the world-wide severe economic crisis could not be foreseen by the commonly used economic growth theory. As those models still suggest infinite growth should be achievable by just creating enough money to trade with. But despite the fact that central banks in USA, Europe and Japan created any amount of cheap and liquid money since, economies did not get adequate growth in return. At least far away from the growth rate classical growth models still predict. Some economists in return now even suggest that macroeconomy can not be analysed mathematically at all. But financial crisis regularly occurs in any country and history. It is far away from being seldom[2]. So any realistic mathematical growth models must include the possibility of financial and/or economic crisis, which happens regularly in economic history.

We will show that this can be done if we just use the main algebraic tools for analysing systems as provided e.g. in physics and engineering. Theories, like e.g. mechanics or electrodynamics, are so called *Field Theories*. Those may be applied to any real life system by using stringent and unique mathematical reasoning. In this paper we will not address the whole of the rich maths known from field theory, but just some sample maths[3] as needed for a very first introduction into the field.

1. The Principle Problem with Classical Growth Models

There exists a lot of economic growth models, one of the more prominent classics is the Cobb-Douglas production function (CDPF, first foundation in 1927) or also the so called AK-Model and uncountable derivations. The problem with virtually all those models but is that they are exogenous models of GDP *Y* in respect to the capital inflow *K*.

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[2] See e.g. the very informative lists of banking and economic crises at the Wikipedia: https://en.wikipedia.org/wiki/List_of_banking_crises & https://en.wikipedia.org/wiki/List_of_economic_crises

[3] Full maths and details of the derivation may be found in literature [1-5]

As they are of the principle kind Y=f(K, parameters ….), which means they are a singular function for Y with an arbitrary inflow of K coming miraculous in any amount from the "outside" of the economy as just an additional input parameter. This assumption reflects the general notion that the capital market's follows its own laws and must be considered from the real economy separately[4]. E.g. the CDPF $Y = A \cdot L^a K^b$ states that growth must occur if just capital K or labor force L is increased. Furthermore the formula has three calibrations parameters (A,a,b) which allows to mimic any behavior as an inter- or extrapolation of real data. Other models of this kind Y=f(K, parameters….) will work in a comparable way.

Mathematically such classical models are statistical extrapolation formula, which may work predictive more or less well just within very narrow limits. From the view of theoretical physics, such kind of formulated assumptions but are not considered a "model" at all. We will give a short explanation how physical model building works in principle in the next chapter.

So physical model building is based on collecting the main influences (forces) which constitute a differential equation which describes only the possible changes of a given function like e.g. Y. Integrating such equations then will give one a principle class of function which solve the problem.

To build up such differential equations but one has to do economic thinking just from the start on, as it is crucial to see and incorporate the main economic forces which influence the evolution of GDP and Capital evolution. No one could doubt that the real economy leads to money and, other way around, money is needed for real economies to be achieved. Both obviously are in some way dependent on each other. A real life model thus must reflect this principle feature. This means that from pure mathematical reasoning the constituting system of differential equations must be coupled, not singular. The results of the algebraic research, which will be again of the kind Y=f(K, ...) and K=f(Y,....) but then are an outcome of a stringent derivation on first principles and differential/variational calculus.

The full maths of the theory may be found in publications [1,2,3,4,5][5]. In this paper we will just show some of the main outcomes of the calculations and reduce maths to a minimum[6].

2. Some Remarks on Classical Physics: Mechanics and Field Theories

First, we will do a short introduction into the mathematical principles of theoretical physics, which started with the works of Isaac Newton (1642-1727). Newtonian mechanics was the starting point of theoretical physics. It means from now on physics and engineering got to be a stringent mathematical science based on the fundamentals of nature. Finally it enabled mankind to invent and build modern goods as e.g. cars, planes and rockets. It was not a field theory from the very beginning[7] (1687), but needed a lot more theoretical development to be finalized. Other prominent scientists like Gottfried Wilhelm Leibniz (1646-1716,

[4] Sometimes called "dichotomy" in economics. It simple words it reduces the question on finances to just "Do you have money, or don't you? If not, get some". This approach may be suitable for microeconomics, but not for macroeconomics, where the evolution of financial and the real market confined and coupled in a (at least approximate) closed national economy.

[5] References [3] (short paper) and [5] (full paper) are freely available via Web.

[6] The simple maths needed is numbered in brackets (.)

[7] Newton's first book was published in 1687, two updated editions followed in 1713 and 1726.

mathematical development of differential[8] calculus), Joseph-Louis Lagrange (1736 –1813, Lagrangian mechanics developed from 1772 to 1788), or William Rowan Hamilton (1805–1865, Hamiltonian mechanics first formulation in 1833) did crucial work to develop it to a real consistent field theory, e.g. formulating classical mechanics in terms of energy. The final “brick in the wall” was found by Emmy Noether (1882-1935) not before her famous publication of 1918. Noether's theorem explains the connection between symmetries of a system, which may be called also invariants, and the resulting conservation laws and equations of motion through variational calculus.

For easiness we start with the old Newtonian style of physics which we will apply to economics in the next chapter. The basic three statements of Newton where:

1. Any object will stay at his motion until there is a force which can change its state of motion
2. Any force will have a counter force $F = -F_c$ such the balance is $F + F_c = 0$
3. The change of movement can be calculated with the differential equation $F = \frac{d}{dt} p$

Although the main formula (3) $F = d/dtp = d/dt(mv)$ seems to be quite simple, it gets very much complex in reality. As both sides are three-dimensional sums which may add up on infinite numbers of mass points *m*, e.g. in gases or fluids. It will make the equations very rich and sophisticated enough to do modern physics until today. In simple terms, Newtonian laws on nature state

1. nothing comes from nothing
2. intrinsic forces must be balanced
3. the sum of all acting forces leads to the change of the system in time (“equations of motion”)
4. the equations of motion can be derived through stringent differential calculus

3. Mechanics of the Macroeconomy

We start with some econometric statistics:

The first graph shows the statistics of the Bundesbank for Germany from 1950 – 2010 in terms of *Y* (GDP) and the sum[9] of all assets of the financial institutions *K* (CAP) and also the sum[10] of all loans (LOANS) given to the non-financial real economy.

To keep things feasible for algebraic integration, we must confine our basic system to the two most[11] important equations of motion: *Y(t)*, the GDP, and *K(t)* the Banks Balance total sheet.

[8] Newton’s handling of differential calculus with so called “monads” was indeed much too clumsy in dealing with its mathematical richness. Today only the Leibniz calculus survived.
[9] Time Series OU0308 provided by Bundesbank: Banks balance sheet total / All bank categories.
[10] Time Series OU0115 provided by Bundesbank: Lending to domestic non-banks /Total / MFIs/All categories
[11] More equations of motion (EoM), like for loans and/or labour can be calculated. As but additional complication will make algebraic integration of the constituting system of differential equations horrific complex, we best start with the most simple but non-trivial model. This is, at least the two most prominent EoM must be taken (less would be trivial and gives no interesting information), which are Y and K as those represent the outer border of a closed national economy best.

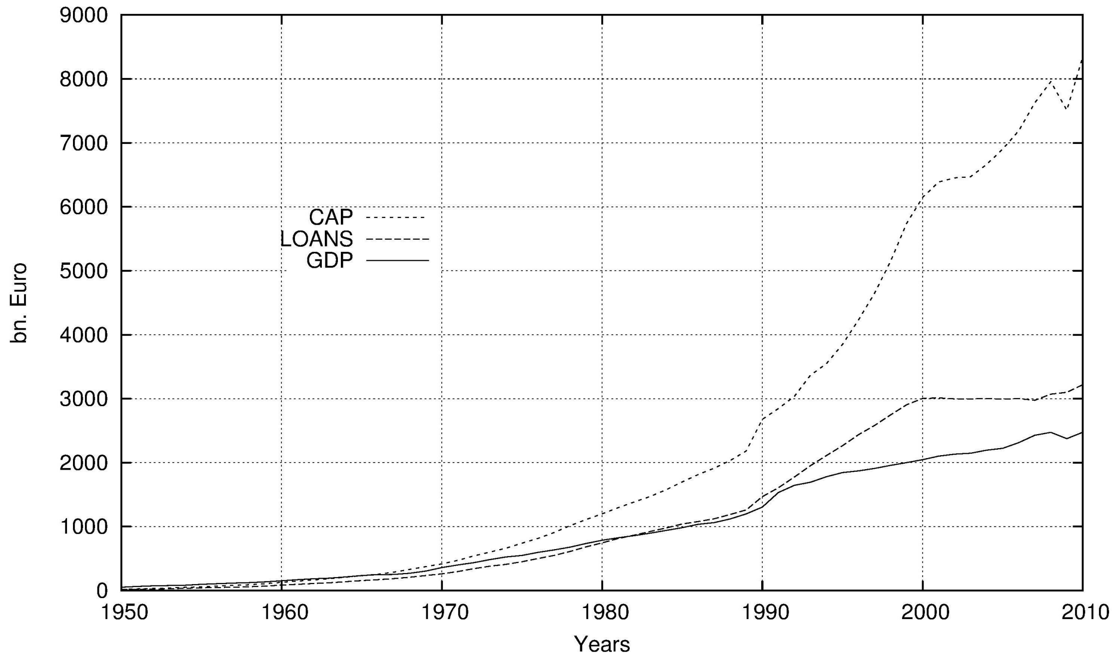

Fig 1A: GDP Y and the sum of financial assets [CAP] K in the FRG from statistical data of the Bundesbank and the Statistisches Bundesamtes (destatis.de). Data from years 1950 to 2010 in billions of €. The jump in GDP around 1990 arises from the sudden inclusion of the people of the GDR which added significantly to the population of the FRG.

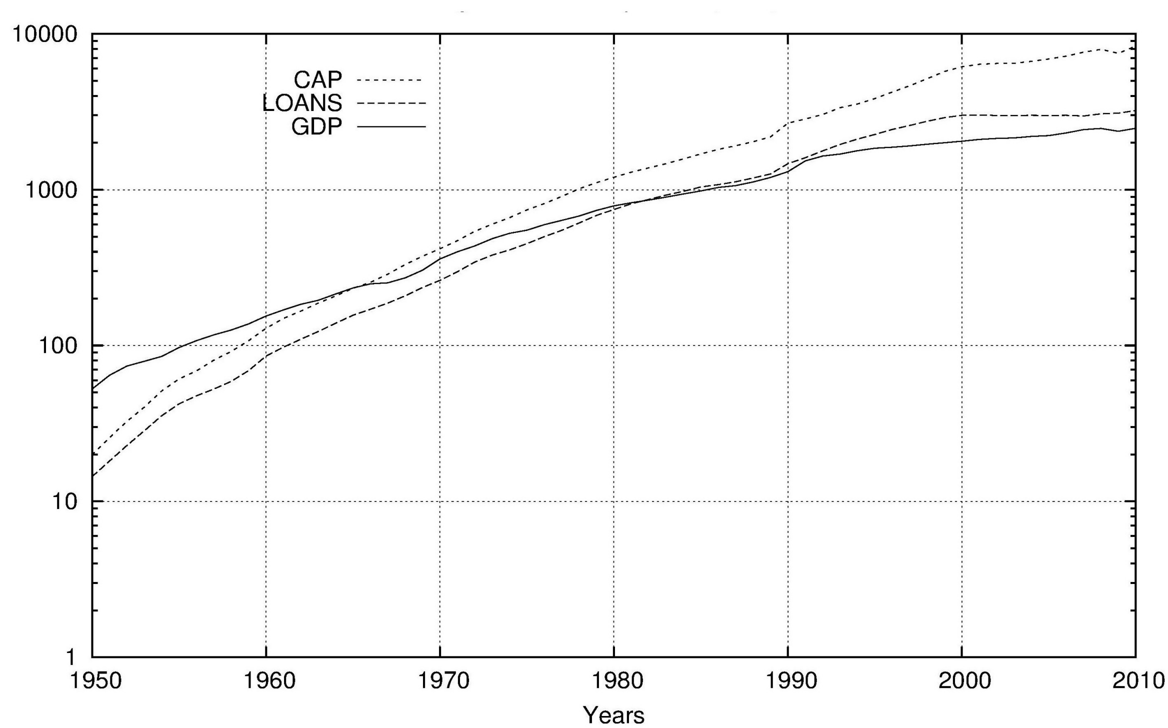

Fig 1 B: Same data in logarithmic plot in y-axis. In the logarithmic plot one can see more clearly the fact, that in 1967 the financial capital stock grows beyond 100% of GDP, and from 1982 even the loans to GDP begin to exceed 100% of yearly GDP. The time-dependent slopes of Capital and Loans both are but unfortunately always steeper than GDP growth.

Also we need the amount of loans given to the real economy, which clearly is a subset of K. It is is the part of financial industries we will call commercial banking (CBB: commercial banks business, consumer banking, here also LOANS[12]. It is the part of the total balance sheet that is directly working in the real economy. The rest up to *K* (which then is BoB=K-LOANS) we will call "Banks own Business" (BoB). It is the financial business with papers of all kinds, sometimes called Investment banking. At a first view we already see that BoB gets more and more strong with time, outlining CBB and also GDP, up to two times and more finally.

4. The Mechanical Model

We demonstrate how to find the algebraic solution to the most simplest non-trivial differential system of the macroeconomy. It was published[13] first in "Die Bank" [1] in 2011. Following classical mechanics $\frac{d}{dt}p = \sum F$ we sum up on the right-hand side the acting forces for growth $\frac{d}{dt}Y = \sum acting\,forces_Y$ and Capital gains $\frac{d}{dt}K = \sum acting\,forces_K$. We thus can constitute the following principle mechanical system:

$$\frac{d}{dt}Y = b_0 + p_B Y + p_Y K \qquad \text{and} \qquad \frac{d}{dt}K = a_0 + p_S Y + p_K K \qquad (1)$$

The functions[14] $a_0(t)$ and $b_0(t)$ represents arbitrary external[15] contributions into the system. In typically quasi-closed economies they play no big role and may be omitted with $b_0 \equiv 0 \equiv a_0$. The GDP *Y* has two intrinsic sources for growth: one will occur by population growth $p_B \cdot Y$. The main force for growth but is induced by capital inputs (loans), which is some proportion auf the total sheet $p_Y \cdot K$. Capital gains in the bulk have two main sources, which are savings $p_S \cdot Y$ by firms and individuals out of GDP, and secondly through capital gains by the interests rates $p_K \cdot K$ of assets.

The terms $p_K = -p_Y$ are counter-equal, such that the financial balance $p_K + p_Y = 0$ rules. It says that what is given at a special time instant to the economy, fails at the capital

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12 Loans to the economy, like labour in the realm of real goods, play an important role within the main equations of *K* and *Y*, as it rules $LOANS = a \cdot K$ and $Labour = b \cdot Y$ with *a, b* some time-dependent percentages of K and Y. It must be remarked here, that both and more may be calculated self-consistent in the full theory. As every additional equation of motion will increase the complexity of the interdependent system of differential equations dramatically, this will be done in ongoing research. A coupled system of four differential equation (*Y, Labour, K, Loans*) instead of two will possibly make the system analytically unsolveable, while it will still be numerically integrateable. In terms of a closed system (see remarks following) but it won't alter the main equations of motion *K* and *Y*, as those additional EoM are just subsets of *K* and *Y.* But it will give great analytical insight into the development of Loans and Labour over time.

13 English translation of the German article in "Die Bank" can be found at reference RWER [3].

14 Like Y and K also all the parameters p_x may be time dependent functions in principle. In simple cases but one or another may be set constant or approximated with e realistic function. The $p_x \in [0,1]$ are written as percentages with $1 \equiv 100\%$.

15 External or exogenous contributions may stem from unbalanced trade or from unbalanced capital flows from/to abroad. Also creating money "from nothing", e.g. just printing money by self purchase of government bonds.

market at least for the same short instant (*d/dt*) of time. We call the resulting rate p_n the "*net business rate*" of financial institution. This is as it reflects the relation of CBB and BoB in financial business, which changes slightly over time and can be derived from Bundesbank data. Mathematically p_n describes the proportion of total banks business which goes to the real goods business by loans to the economy (commercial and retail banks business) in terms of the proportional overall interest rate. If it is negative it means most financial business is done with the real economy. Its actual value can be deduced from the long-time series of Bundesbank for CBB and banks total balance sheet.

We may cite here Morrison[16] et al: "*Investment banks are changing fast. Forty years ago the industry was dominated by a few small partnerships that made the bulk of their income from the commissions they earned floating securities on behalf of their clients. Today's investment banks are huge full-service firms that make a substantial proportion of their revenues in technical trading businesses that started to attain their current prominence only in the 1980s. The CPI-adjusted capitalization of the top ten investment banks soared from $1 billion in 1960 to $194 billion in 2000. Between 1979 and 2000, the number of professionals employed by the top five investment banks (ranked by capitalization) rose from 56,000 to 205,000…. Although investment and commercial banking were separated by the 1933 Glass-Steagall Act, commercial banks started in the 1980s to encroach upon some investment banking activities. The establishment in the late 1980s of so-called Section 20 subsidiaries allowed several major commercial banks to underwrite corporate debt issues provided this activity accounted for no more that 20% of the banks' gross revenue*."

The described effect can be seen in any modern economy, which is that the realm of investment banking has overtaken the commercial and retail banks business by far until today. Changing financial techniques from giving simple credits to selling financial instruments like securities, shares, insurances etc. have a lot of benefits for financial business. Anyway it is to get better revenues. The need can be seen simply at the graph Fig 1. With time the GDP gets over saturated on loans, the demand for further credit declines which is accompanied by a steady growth of money supply K. Thus interest rates will fall as those are the "price" for alien money which always is regulated by supply and demand as it rules for any product. In effect the shift from mainly CBB to mainly BoB is a financial need for banks to not dry out on revenue but to maintain higher interests.

But this business model won't last eternal. As an example we may take collateralized debt obligations (CDO) which effected very much the subprime crash which ignited the actual financial crisis. CDO were used by commercial banks to refinance their business. This is, as credits given to someone will diminish the banks' ability to give new loans as far as the old ones are not yet repaid. Packing such credits to a CDO but makes them sellable financial products. If one finds a buyer one gets cash in return which again can be used to give new loans in CBB business. So large amounts of mortgages, even risky one's, where packed into CDO's which were sold to investment banks and investors even abroad. The risk was transferred away from CBB, which now could give more loans, but on a finite market of good borrowers. Finally only the worst borrowers were left, but in confidence to ever rising house prices, those people were imposed to credits which they should pay by virtual wins of ever rising prices in the future. On finite markets but such a perpetual mobile is doomed to crash.

[16] Cited from Investment Banking: Past, Present, and Future, Oxford University Press by Alan D Morrison, Saïd Business School, University of Oxford and William J Wilhelm, Jr., McIntire School of Commerce, University of Virginia, p8, p18. https://gates.comm.virginia.edu/wjw9a/Papers/JACF%20Morrison%20Wilhelm%20Final%20version.pdf

As such principle problems were already known since the 1930ies, the Glass-Steagall Act forced a strict divide between CBB and BoB. It may be argued that, even if this Act would be in force until today, it would not have prevented the crash. The crash but would have occurred somehow later and Glass-Steagall Act may have prevented the flash over to CBB and rising of national debt through the following bailout's to prevent the whole financial system from collapsing. On the other hand we may argue, what could be an alternative to investment banking at all? Without it CBB would run out on profits after some decades of big business. It then could only be financed as a state task continued from tax revenues. The net business rate p_n mathematically translates exactly this principle behaviour of the financial business, which is in the overall bulk a smooth transition from CBB to BoB.

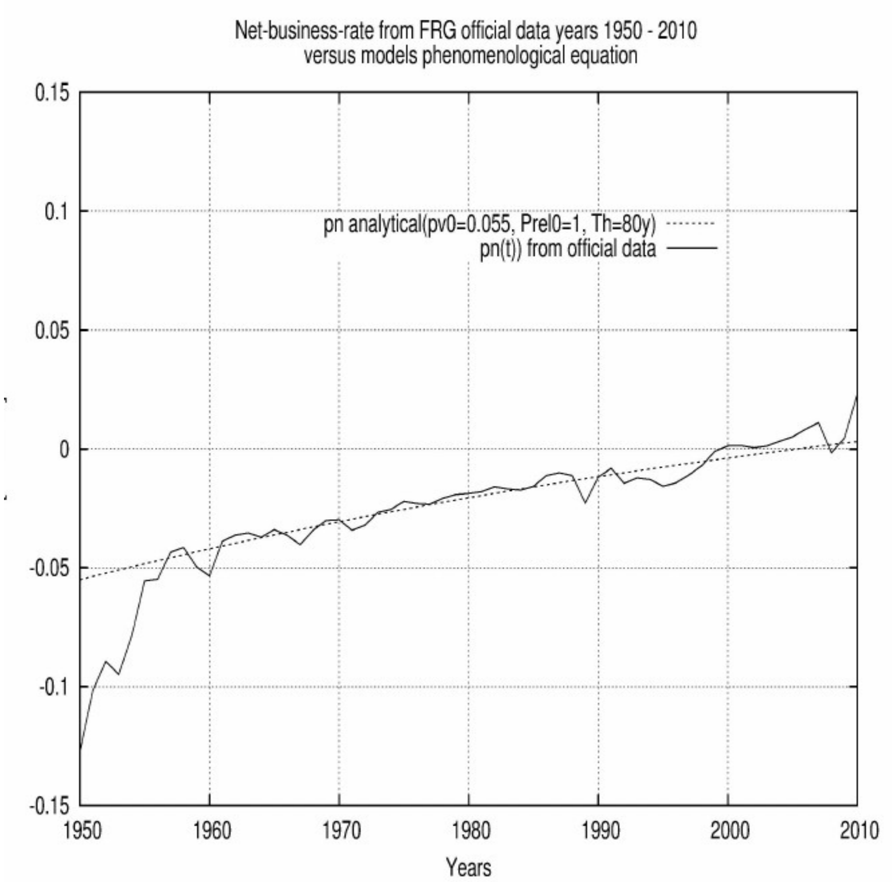

Fig 2: The net business rate from official data (solid line) and approximated by our smooth exponential (dotted line) years 1950 – 2010.

In Fig 2 we see (solid line) the net business rate as derived from official statistical data of the German central bank Bundesbank. The sharp drop until around 1955 results from the fact, that after the war in 1945 Germany was a completely destroyed country. Thus in the early years virtually every money available was given into the real economy by CBB with interest rate up to 13% and thus great returns for CBB. After this decade of rebuilding the economy but the usual banking business could be established as a mixture of CBB and BoB. In 2000 finally BoB overtook CBB. In the average this process but is a smooth process which can be approximated by a slightly increasing function towards investment banks business.

5. Analytic Integration of the Model

For algebraic integrability p_n must be approximated by a smooth exponential[17] function, which can be seen in Fig 2 as a dotted line. Further, again for *algebraic* integrability we can set the population growth roughly $p_B = 0$ to zero for Germany. Also the German savings quota was nearly constant at 10% over the decades, so we may set $p_S = 0.1$ constant[18]. Thus we get the simplest possible set of model differential equations as

$$\frac{d}{dt}Y = -p_n(t)K \quad \text{and} \quad \frac{d}{dt}K = p_S Y + p_n(t)K \qquad (2).$$

The differential system (2) is a so called "*starting value problem*[19]". The integration delivers a function $\Phi := p_n(4p_s - p_n)$ which has the dimension of a frequency and is crucial for the kind of growth. For mathematical details we refer to reference [5]. The resulting equations of motion from algebraic integration may be roughly simplified as

$$Y = a(\Phi,t)\cdot \exp(g(t)\cdot t) \quad \text{and} \quad K = b(\Phi,t)\cdot \exp(g(t)\cdot t) \qquad (3)$$

where we notice some factor functions *a* and *b* with some very complex time dependencies, multiplied by an exponential growth $\exp(gt)$ with but an also time dependent argument $g(t) = \frac{p_n(t)}{2}$. The Newtonian solution thus looks structural like a classical steady-state model. Such a steady growth model was used e.g. by the IMF and others. Those read simplified as $Y = a\cdot\exp(g\cdot t)$ and $K = b\cdot\exp(g\cdot t)$ with but constants *a, b, g. This makes a large difference.* The time dependent factor functions are harmonics with frequency Φ, which now modulate the otherwise[20] seemingly steady growth. This stems from the coupling between *K* and *Y* which is not present in classical models. From algebraic integration of the basis equation one gets a characteristic time T_C of an economy, which in the case of Germany equals about 50 years. From starting point 1950 thus giving around year 2000 as the entry to the limits of growth and begin of the realm of financial (debt) problems. It crucially depends on savings and on the net business rate of financial institutions.

6. Numerical Integration

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17 The detail for the derivation of p_n see e.g. ref. [5]. In principle it is a smooth and close approximation to the real data evolution over time which can be treated for algebraic integration. One may see the algebraic function as the dotted line in fig.2.

18 In numerical integration we may easily respect any $p_B = p_B(t)$ and $p_S = p_S(t)$ as a time dependent function. For an algebraic integration but any additional time-dependent function makes calculation much more complex. Setting them constant, like it is indeed a good approximation for Germany, but will deliver us the simplest possible model which reflects just the most fundamental basics of the interaction between *Y* and *K* in any country.

19 It means that we need just the two single point starting values $Y_0 := Y(1950)$ and $K_0 := K(1950)$ to adjust the origin of the economy. The rest then just is the time-dependent change which follows only rules given by the conditional differential equation (2).

20 We may shortly discuss the difference to classical models: As from experience obviously *Y* and *K depend on each other*, so the equations must be coupled and balanced. The simple Newtonian approach naturally generates such a system. This is mathematically unavoidable but due to the dichotomy claim seldom done in economics. Another crucial point is, that *the whole of all assets* is included in the calculation, not just e.g. loans to the economy or any other arbitrary part of *K*. Ad hoc there is no reason to exclude any part of the financial economy form the overall calculations.

Algebraic integration is much more fruitful than numerical integration as it conserves all input parameters and dependencies from the start to the end of the calculations, which get lost in pure numerical calculations. The great advantage of numerical integration but is that virtually any complicated differential system may be integrated, finally limited only by available computer power. We now integrate this starting-value system numerically to find out if it matches the real data. Which is the equivalent of doing experiments in physics: It checks out if theoretical assumptions invested in the constituting differential equations are reflected in the measurements[21]. This results in the following graph:

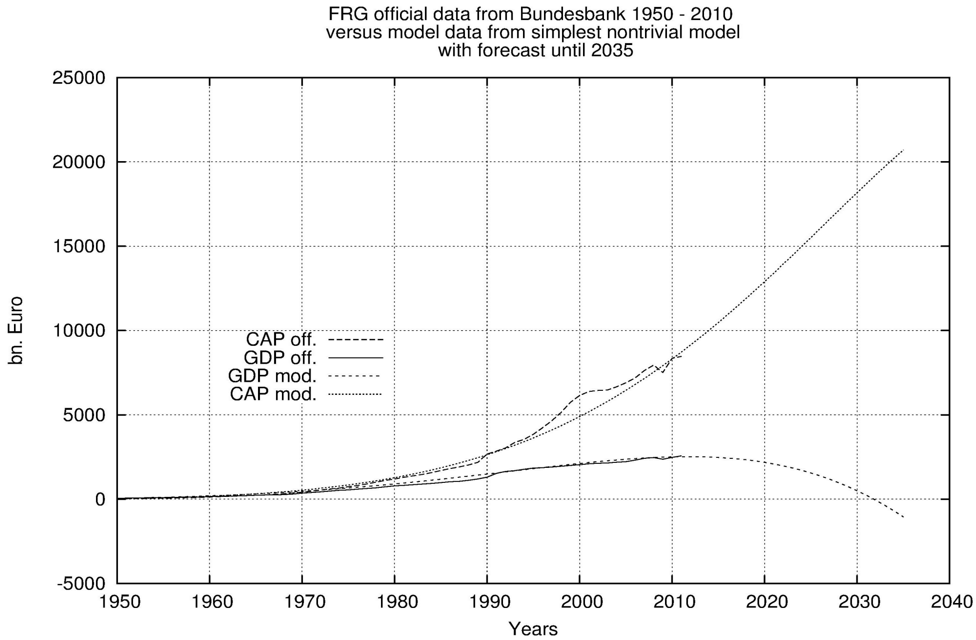

Fig.3: Numerical Integration of the Simplest Model as Used for the Algebraic Solution Too

It must be pointed to the fact here with rigor that it is a starting[22] point value problem. The only things we invested here to feed the mechanical solution are the starting values of K and Y in the year 1950 and the known relation of CBB to BoB financial business p_n . The evolution of the equations of motion until year 2010 follows automatically. Thus it tells us is, that the evolution of GDP *Y* and the total banks balance sheet *K* depends critically just on the interests mechanics of the financial market.

As we do a numerical integration we may now easily put in more time dependent data, to show how it influences the outcome. In the Fig 4 we see the effect of population growth, simply by now using $p_B = Data_{official}$ official data instead of the rough assumption $p_B \equiv 0$.

[21] Which is a must. Theoretics in physics is always such a circle of theory and experiments. From experiments, which are anyway statistics, we get a first guess of the constituting forces. From which a first theory follows. Integrating then must lead to coincidence with experiments. With further refinements, and this circle of try and error, a physical theory may be build up until it is finally unique and exact.

[22] This means, that we only introduce the value of the starting point, here in the year 1950, into the equations, this are $Y_0 = Y(1950)$ and $K_0 = K(1950)$. The rest of the calculation is self-consistent evolving from the intrinsic dependencies.

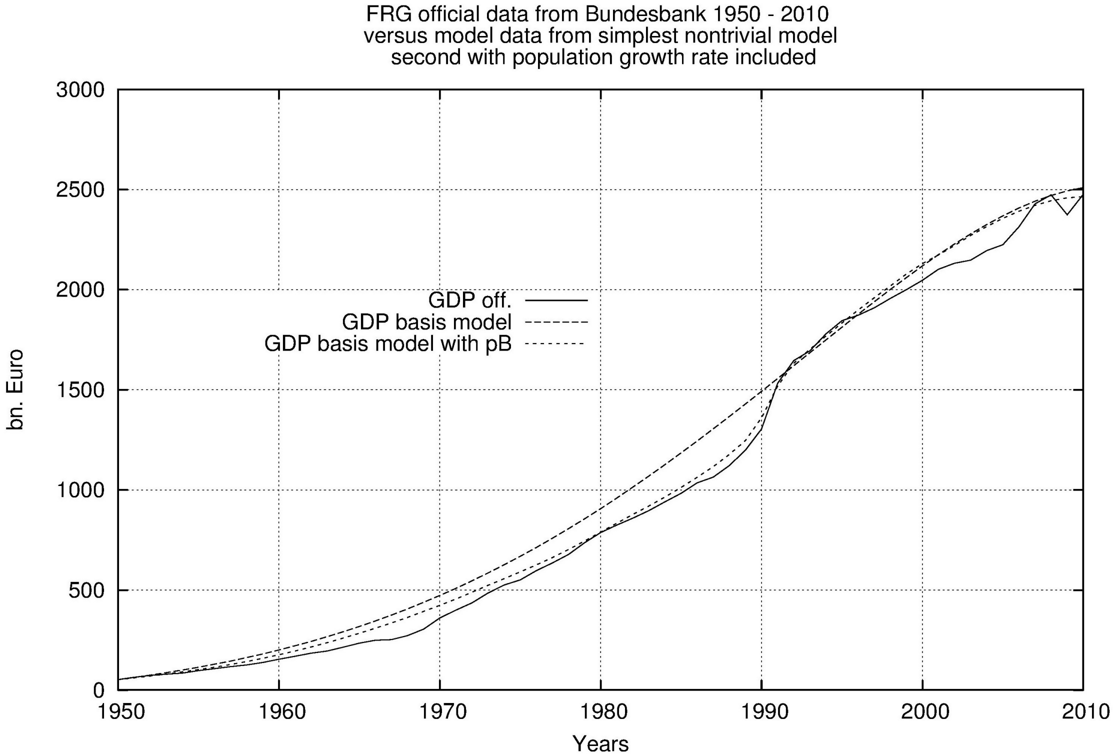

Fig 4: This plot shows three functions: (1) GDP from Bundesbank official data (full line); (2) basis model calculations based just on starting values 1950 (long dotted line); (3) Additional curve (short dotted line) shows the model outcome if known real population growth is included in the integration.

The basic model omitting population growth already shows good agreement (upper long dotted line). Adding the real numbers of population growth makes the model prediction but even more exact and very similar[23] to the real life time lines. The small deviations already left, which may be seen as under-performances of the real GDP growth in relation to an undisturbed theoretical market, can be assign to exogenous influences. These were in the late 1960s the Macroeconomic regulation done by German government under Minister of finance F Schiller based on Keynes' economic models. It created a lot of additional money for the real economy for the price of an unnatural growing of national debt. The breaking of the GDR in the late 80ies and its final inclusion in 1990 may be identified, and finally the start of the Big Business of Investment banking in the late 90ies until today which disturbed GDP growth very much by money from abroad and also e.g. by money creating through central banks even purchase of government bonds.

7. Invariants

To achieve a full field theory of economics, one has to find at least the most prominent invariants of a system. After some calculation we found that the old[24] fashioned Quantity Equation of Exchange, also known as the Fisher equation $MV = HP$ (1911), is one of those principle symmetries.

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[23] It must be recalled, that we neglected foreign exchanges here and exogenous money creation as well for algebraic easiness. Also we should put into mind the fact that statistical data of GDP can never be 100% correct. Within these limitations we can see that the theoretical predicted and real life measured lines are already quite identical in the linear Newtonian approach.

[24] First mentioned in a philosophical way by Copernikus (1473-1543), mathematically stated in 1911 by Fisher.

To achieve this, we may simply rewrite it in usable terms as $\frac{d}{dt}\left(\overline{M}\overline{V} - \overline{H}\overline{P}\right) = 0$, where $\overline{M}, \overline{V}, \overline{H}, \overline{P}$ will be matrices of some larger dimension and thus it gets a macroeconomic balance equation. The matrix elements may contain every trade, financial and real goods, and contracts as well, even world-wide. For practical reasons the analysis must be restricted to dimensions which are algebraic or numerical manageable. With the use of such invariants the linear Newtonian theory may be extended to a full non-linear[25] theory by virtue of Noether's theorem. The smallest non-trivial reduction is the one of two dimensions which reads

$$\begin{pmatrix} V_R & V_{RI} \\ V_{IR} & V_I \end{pmatrix} \cdot \begin{pmatrix} M_R \\ M_I \end{pmatrix} = \begin{pmatrix} P_R & P_{RI} \\ P_{IR} & P_I \end{pmatrix} \cdot \begin{pmatrix} H_R \\ H_I \end{pmatrix} \qquad (4)$$

for the matrices.

It decomposes the economy just in its two main realms, the financial sector I and the real sector R of an economy. In the linear case it shows up again to be the Newtonian linear equations (1),(2) we found in the beginning. It but may be expanded to any[26] number of sectors and products of an economy. This is the realm of future theoretical research.

8. Some Interesting Sample Outcomes of the Newtonian Theory

a) Crisis Indicators

Classical models always suggest that with some additional input of money a tumbling economy should be always be healable. E.g. the CDPF $Y = A \cdot L^a K^b$ tells that the marginal productivity $\frac{\Delta Y}{\Delta K}$ of capital will diminish[27], but never gets zero. From which follows that, if much money doesn't help, one shall try to inject even much more money into the system until finally GDP growth will again take on. But experience shows that this marginal productivity sometimes even gets zero and below. Newtonian solution now shows that this but is the natural behaviour of the economy.

From economic thinking this effect may be seen also. In recent years central banks worldwide made unlimited quantities of cheap money available. Despite some seemingly just temporary effects no general healing was achieved. This practically can be attributed to the fact, that this money mainly was invested in healing and even enlargement of Investment banks business. Instead of the hoped-for effect that the cheap money was put into the real goods industry, thus giving reason for additional jobs and consumption. This made the relation of CBB to BoB even worse. From a bankers view but it makes sense, as investments into an already over credited GDP will give no competitive returns. Definitely buying and selling debt instruments like e.g. treasuries make more microeconomic sense for banks. Especially, as the vicious circle of crisis costs and thus enlarged national debts makes government bonds even more attractive from rising risk and thus rising interest rates. Since

25 See Chapter 21 and following Chapters in [5]. For linear theory see chapter 10.

26 At least a numerical integration can be done up to any higher dimension limited just by available computer power and reliable input data for the starting values. Additional invariants can be found, and every one will lead to new algebraic insight. Of special interest here will be the self-consistently calculation of interest rates and the evolution of labour power.

27 With the usual choice of positive $0 < b < 1$.

e.g. the EU decided to guarantee all government bonds of its states, just the worst national debts with the highest interest rates among them got to be the least risky “cash cows” for BoB.

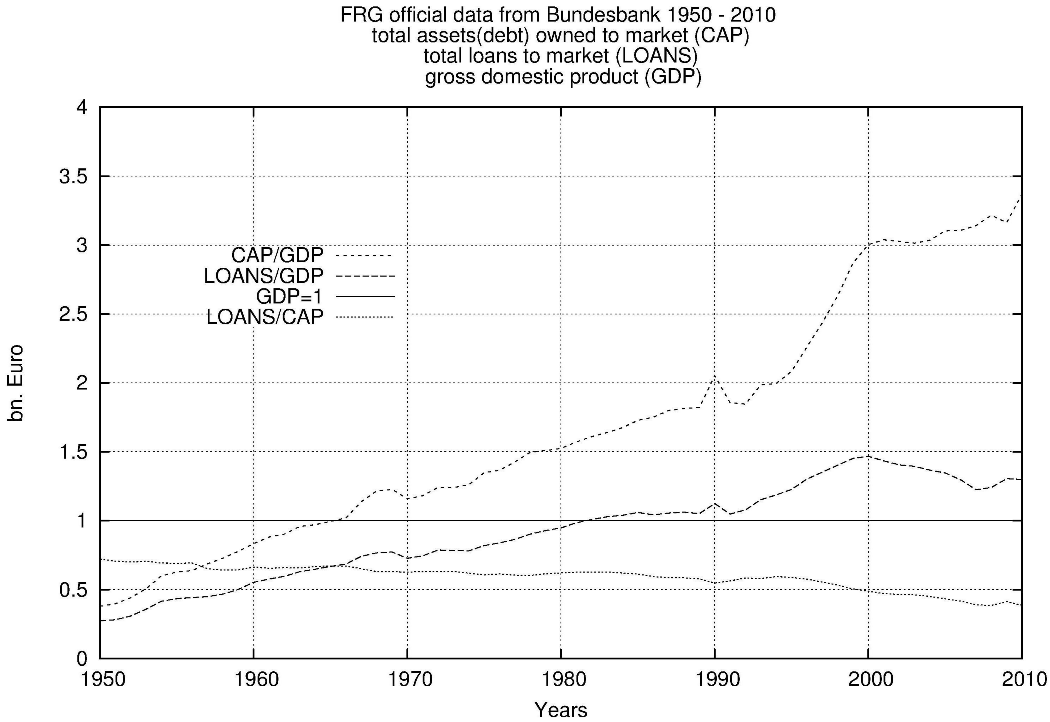

Fig 5: **Fig. 5:** Official Relative Numbers for Germany

Such Economic crisis will occur regularly when growth vanishes. From our calculations we can estimate, that growth will come to stagnation

- if the amount of K gets more than 300% in relation to the (yearly) GDP Y
- if the CBB gets less than 50% of the overall Business of financial institutions
- if Loans to the economy steadily grow up to around 150% of GDP and then starts a steady decline
- if marginal capital productivity tends to zero and statistics show up “crisis slopes”

The dotted line right down at the bottom of fig. 5 is the relation of Loans to the whole of Capital K. It went below 0.5=50% in 2000. The same happened to the whole of all assets which overrun 3=300% in 2000. At the same time the outreached loans to the economy climbed to its all-time maximum of nearly 1.5=150%.

As one knows, since 2010 Germany performed much better in relation to other western countries. One reason seemingly is that the foreign trade balance of Germany doubled

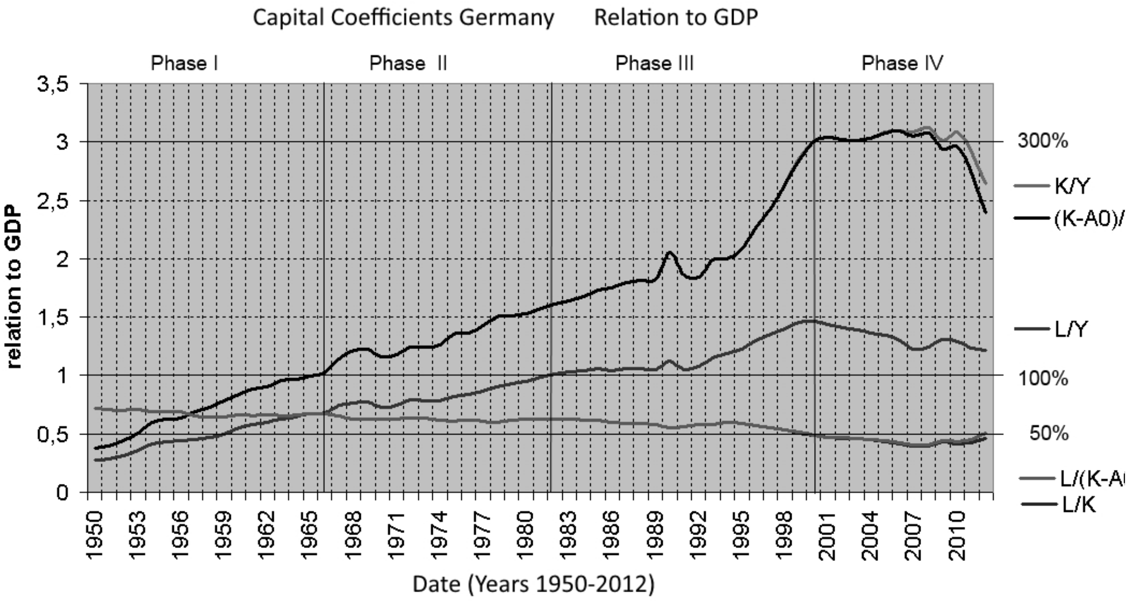

Fig 6: Capital Coefficients for Germany in relation to GDP derived from the official data of Bundesbank for the years 1950 to 2012. K means total of Capital (Assets), L means Loans, Y means GDP, a0 assets debt paid abroad, see equation (1).

since the year 2000, and trade balance surplus got even larger than that of China. Another reason may also be found in the banking sector. So Germany "lost" a lot of money in the whole, which means preferably it went abroad. In effect Germany, at least temporary, went out of the crisis region (fig. 6). As in 2012 the *K/Y*-relation again got below 300% and the relative CBB sector grew again up to greater than 50% of the overall financial business.

Fig 6 shows those real data for Germany from 1950 until 2012. The first phase of economic evolution was a *GDP-driven economy*, which ended around 1967. This is when the ratio of assets to the GDP climbed up to *K/Y*=1=100% (top line). After that the economy but got more and more *capital driven*. Another phase was reached in 1982/1983, as then also the loans (CBB) reached the limit of *L/Y*=1=100% of actual GDP.

This gave some reason for then need of more Investment banking, as the demand for credit from the real economy came to saturation. In the 90ies the high time of investment banking started, which resulted in an accelerated climb of *K/Y*. Around the year 2000 finally the critical amount of *K/Y*=300% was reached. At this time the DotCom-Bubble imploded, and the realm of growth crises was reached. Unlike southern European states Germany but again got into the region of at least moderate growth after 2010, as the crucial 300% line for *K/Y* was left and dropped to around 250% again. If this improvement of economic performance but is durable may be doubted, as it crucially depends on the ability and willingness of German export partners to consume German goods, and on investors belief that high gain assets are better off abroad.

As a consequence, our calculations suggest that a crisis solution seems only possible if less money is on the market available rather than increasing amounts. The share of CBB must be lifted over the 50% mark permanently again. However, achieving this goal is politically far from easy. Especially, since it is the classic opinion contrary that less money can be more. One strategy could be to narrow BoB through regulations by law and taxing revenues high.

This concerns but hits with considerable resistance from international investors. Even a small, and thus in sum ineffective, taxation was therefore difficult to implement in the past. Most even the opposite was the case. With microeconomic arguments of investment bankers more regularly a weakening or even exemption from requirements was achieved instead of necessary restrictions.

But politics may use backdoors to heal the CBB/BoB relation. So simply one could invest the money created by central banks directly into the GDP, e.g. by directly paying for the needed renovations and upgrades of the nation's infrastructure. The effect on national debt would be the same, but the impact on GDP would be enormous as it would convert one into one to growth. And finally, after flowing through the hands of entrepreneurs and workers and their consumption again would end on the total banks sheets just some time later. The bulk effect would be large growth, which but is accompanied by appropriate inflation and higher interest rates too.

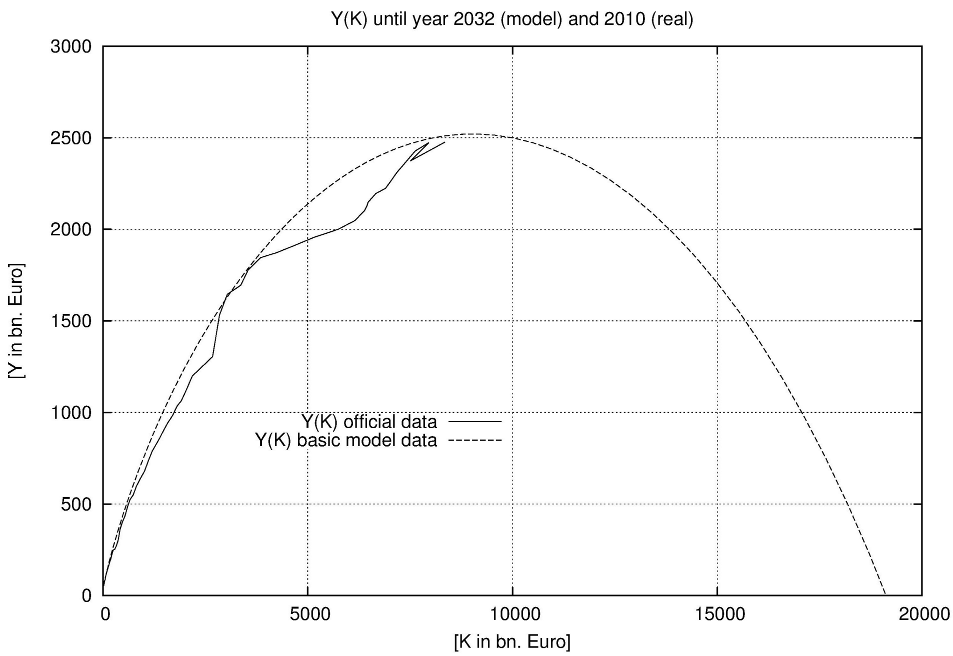

Fig 7: Y(K) from real data of Bundesbank (solid line) and from Newtonian model prediction (dotted line)

Although higher inflation will harm assets in the first line negatively, it won't be that worse for investors. As it is the fact, higher inflation always leads to higher interest rates and finally also to increased growth and level of employment. The biggest difference would be that, in particular, the investment banks would have to earn their money through advertising and competition out of GDP-winners, instead of getting it as a "free meal" from central banks. Either way, it will in any case be a difficult task to promote a shift in the thinking and acting of politics and in the banking business as well.

In fig. 7 we present the real data from Bundesbank. The graph shows the implicit function *Y(K)*. This represents the GDP *Y* not as a function of time, but as a function of the available overall money in the economy *K*. As we may see, the slope decreases with more capital, contrary to common belief, although even the simple CDPF model predicts this. But contrary to that the Newtonian model shows, that the curve would go down again to zero in an undisturbed economy.

But before reaching the "top of the mountain" (*dY/dK=0)* financial politics and central banks get already engaged in the transmission mechanics between *Y* and *K*. We may see the resulting effect of money politics as those "*crisis slopes*" at the top of the curves, which build up since the 2000ties. This is as the injection of money (which is the right handed direction of the curve) doesn't result in an appropriate growth (which would be an upwards move of the curve). While temporarily a little growth may be achieved, a left and downwards movement occurs, which means a recession in growth and a loss in assets too.

We may also see that the effect on exogenous money (created from central banks or an inflow from foreign investors) regularly results in an under-performing of the economy in respect to an undisturbed closed economy. This is as such foreign money mostly went into pure investment banking without or only few CBB. In the last consequence this gives juts reason to a capital drain out of the real industry through interests into the financial industry. Through substitutional effects this results in the observable under-performance.

We must admit that any asset on one hand is debt through banks balances on one's other hand. Finally, with the state as a lender of last resort, any interests have to be paid for from the real economy in the final bill, as substitutional effects transfer it to the GDP. So taxes and fees are preferably charged by the working population. Also any investment product in the last instance must be sold to a private small or large private investor, where it substitutes with consumption or investment into the real industry.

As capital intrinsically grows stronger in the long term we face the problem that at a given time there will be an overhang of assets. Which have ever growing claims on debtors which but are running out in the private field. At last the state has to bail them out to prevent the system from crashing. But just printing money by central banks, giving it mainly to Investment banking instead of directly investing it into the economy, will not help in the long run. We may notice the resulting effect as those Y(K)-"crisis slopes" we may see in any country with such a problem.

b) The Role of Inflation

Analysing the system in terms of field theory shows that inflation is a strictly non-linear effect of the economy. It cannot be handled with the simple linear Newtonian approach (2) alone. Integrating non-linear differential systems is a quite hard task, exact solutions are not easy to find if at all. But physics provides a lot of approximation tools to get at least linearized[28] solution close to the exact one. We present just some outcomes here, which result from quite complex formulas[29]. As an over-the-thumb rule we get roughly

$$I_c(t) \approx p_w \cdot (1 + p_w) \quad (5)$$

where p_w is the actual rate of GDP growth. The essence is, that in average the core-inflation

28 Such an analysis was done e.g. in chap. 18 of [5].

29 We will not present the very complex maths from the non-linear analysis here. In the graph we used the already approximated rule $I_c(t) = \left(d_t Y^2 / Y + d_t K d_t Y^2 / Y^2\right) \cdot t + d_t Y + d_{tt} K / Y + d_t K$ as in (18.16) of ref. [5] for the predictive CPI in comparison with the measured one.

$I_c \geq p_w$ will be a little bit greater[30] than GDP growth. In general we may also conclude from this analysis that usually the inequality

$$p_v \geq I_C \geq p_w \qquad (6)$$

rules.

This means, that typically the *average* returns over all financial assets p_v will be slightly greater than the core inflation I_C which but is slightly larger than the growth p_w . What mechanical analysis shows thus is, that average interests rate, inflation and growth are strongly correlated with each other and roughly have the same order of magnitude.

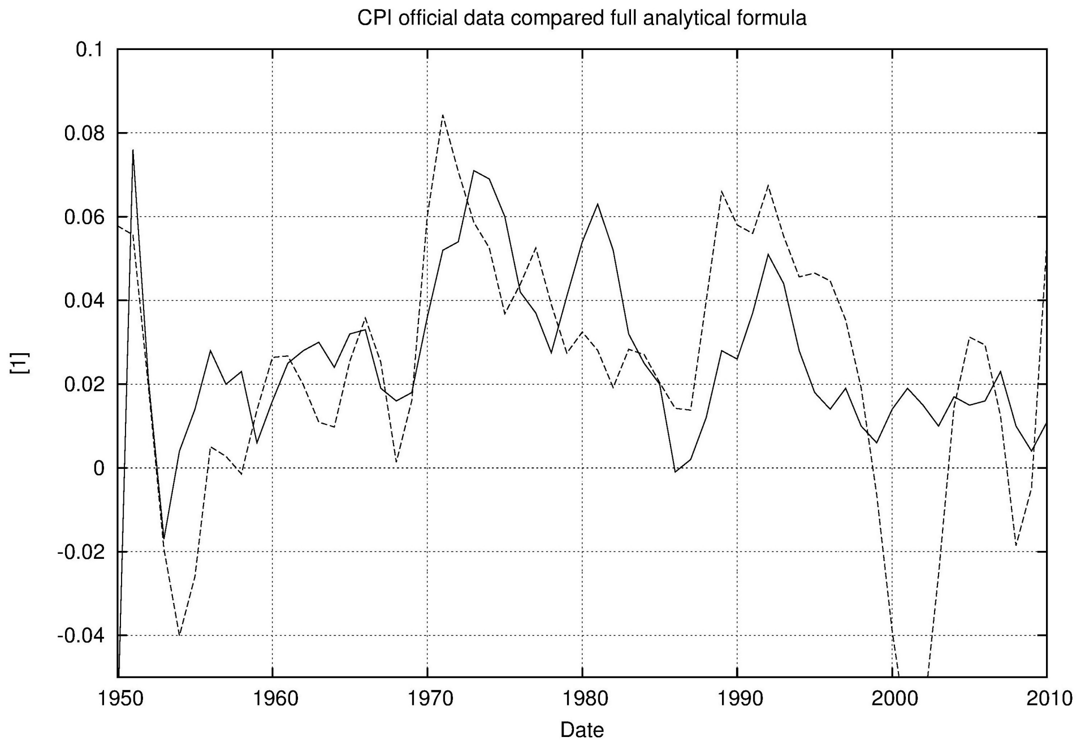

Fig 8A: Modelled and official CPI. Model data for CPI were calculated using official data of Y and K

30 E.g. for actual growth rate 5%=0.05 we get roughly a core inflation of 5.25%.

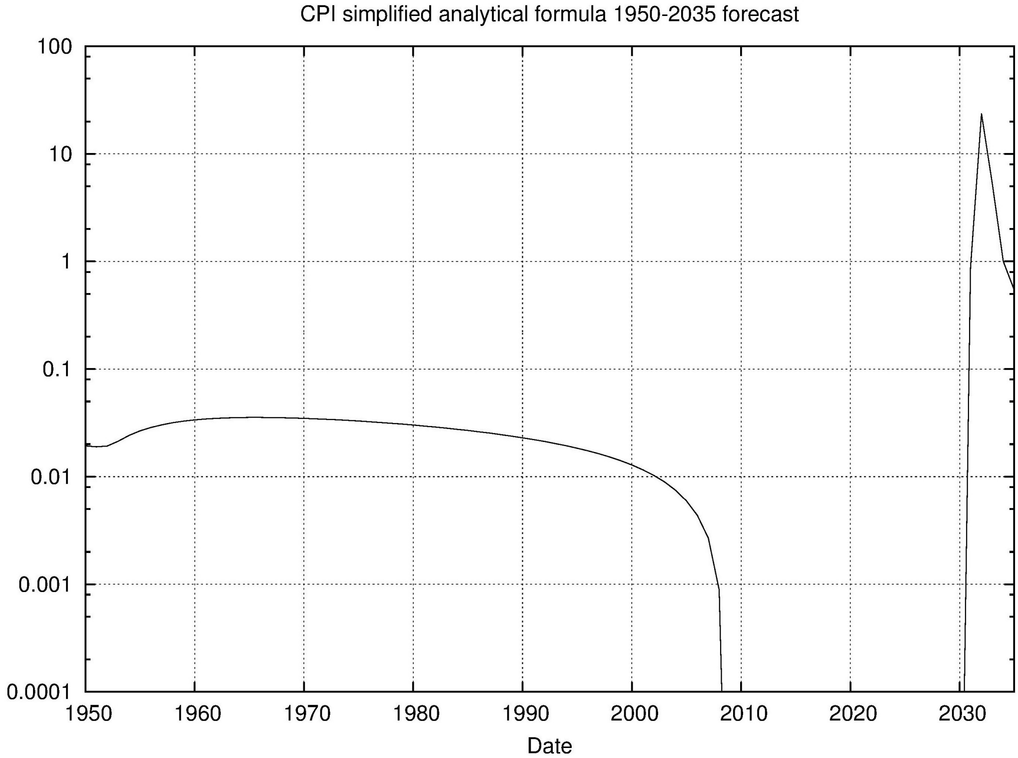

Fig 8B: Modelled CPI in logarithmic scale.

The term "core"- inflation means the average inflation over all products traded in an economy. So it differs from official CPI in some way, as CPI usually is defined by a very special consumer-basket. What we but see in fig. 10 is that the theoretical calculation, here based on the more exact equation (18.16) of ref. [5], fit the official CPI from statistics already in good agreement.

The larger deviations in fig. 8 A can be related to external shocks. This is exogenous money, like the DotCom-Bubble around year 2000 and the inclusion of the GDR around 1990. The theoretical curve is but only disturbed by the DotCom bubble very much. This is because in contrast to the GDR inclusion the DotCom-Bubble was only related to a large transfer of foreign money and papers, not directly to GDP. Thus giving rise in K and its derivatives, but had no direct effect on changes in Y.

The graph fig. 8 B is the principle function for the core inflation in logarithmic scale as it would show up in an undisturbed model economy. It shows that a closed system of the German economy would face a long phase of very small inflation and small interest rates, which could hold on for about 20 years (right hand plot, years 2010-2030). In the end but a hyperinflation could happen (around 2030-2032, with inflation rates up to 3000%). Hyperinflation would occur over an ever growing money supply hitting an ever more shrinking GDP. This scenario but has the premise that politics and central banks would do nothing to help. This is regularly not the case, as with stagnating growth central banks already did a lot of exogenous work to the economies. But it is the players are not clear whether the applied methods are helpful, lead to nothing or even accelerate the collapse. For predictiveness such external interventions could be incorporated into the equations using the terms $a_0(t)$ and $b_0(t)$ by feeding them with the actual politics measures.

c) National Debt

An old miracle are ever rising national debts. Although there are large differences in the relative amount of such debts, what they have in common anywhere is, that they are notoriously growing in the long term even if there is serious political will to reverse this trend. On the other side there is the fact that large or low national debts are seemingly not directly matched[31] with low or large GDP growth. Analysing national debt in respect to the Newtonian model now shows up some hints for a reason:

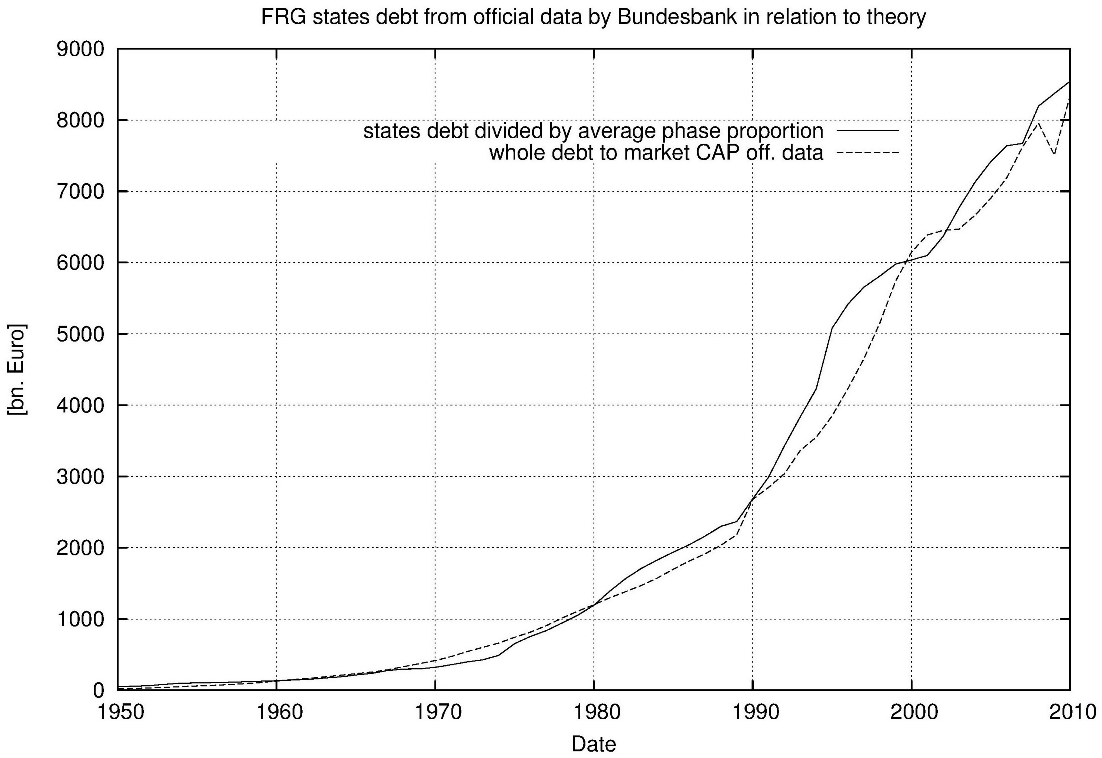

Fig. 9A: National debt (times 5) of Germany official numbers (solid line) against the Sum of all assets in the financial market (dotted line, also official data).

[31] Even countries like Japan with record burdens on national debt (more than 200%) didn't perform that bad. While other countries, e.g. Spain before the subprime crisis (around 60%), despite low national debt didn't perform much better. Not nearly three times better as one would expect.

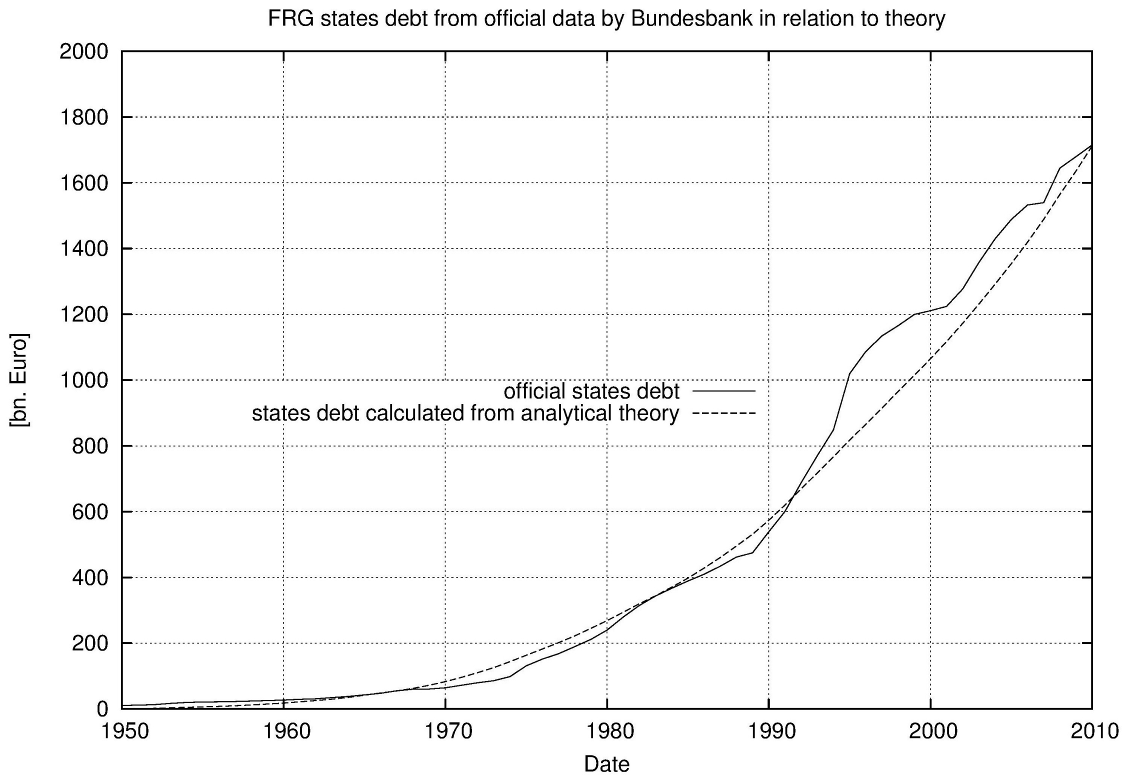

Fig. 9B: National debt of Germany official numbers (solid line) compared with the integrated sum (ref. [5] 17.13) of Germany's savings assuming an average interest rate of 3%.

If we compare the sum of all assets in the banking system *K* to the national debt of Germany, we see the astonishing effect that it is practically an all time share of around 20% of *K*. Which means the curves of National Debt times 5 is practically the same as the long time series of K (fig. 9 A). It clearly shows that the German national debt grew with the exact same slope as the overall private debt. In other words, there is seemingly nothing wrong with the ever rising German debt. It just strictly reflects the financial market evolution. This may be not astonishing if we remember that the state is the largest investor and employer of the nation (public expenditure quotas are typically around 50%) and also the largest single debt holder of last resort. Why the state should, in principle, behave very differently from the private investor?

In fig 9 B we plotted the national debt against the sum[32] of the German savings from 1950 to 2010, integrated assuming an average yearly interest rate of 3%. Astonishingly the savings match practically exact the national debt. The reason behind this fact seemingly is, that bad assets always must be bailed out by the state. This is not only the fact after Lehman 2008 with central banks buying every bad asset to save banks from collapsing, it is also the fact with small money savings. This is as for such small cash business banks expenditure often exceeds income. To minimize expenditures banks mostly transfer the bulk of such savings to secure income assets like government bonds. In the long term average this results in a comparable national debt. In other words one could say that national debts in the bulk are just the savings of the average people.

For more details see Chapter 17 in [5] where one may also find the *substitutional rules for state expenditures* and incomes in a closed economy by virtue of the full Fisher equation.

32 See equation (17.13) on page 49 in reference [5] for the mathematical expression of this sum.

d) Economic Globalization

Since the 1980ies economic globalization has been expanding very much. Despite the common belief it should effect everybody world-wide in the same positive way, Dollar and Kraay (2011) stated[33]:

"*Per capita GDP growth in the post-1980 globalizers accelerated from 1.4 percent a year in the 1960s and 2.9 percent a year in the 1970s to 3.5 percent in the 1980s and 5.0 percent in the 1990s. This acceleration in growth is even more remarkable given that the rich countries **saw steady declines** in growth from a high of 4.7 percent in the 1960s to 2.2 percent in the 1990s. Also, the non-globalizing developing countries did much worse than the globalizers, with the former's annual growth rates falling from highs of 3.3 percent during the 1970s to only 1.4 percent during the 1990s. This rapid growth among the globalizers is not simply due to the strong performances of China and India in the 1980s and 1990s - 18 out of the 24 globalizers experienced increases in growth, many of them quite substantial*."

Newtonian mechanics give some hint to the reason of this effect. In principle our system can be extended to the entire world[34] economy by a large-dimensional system of differential equations. We will show here a small sample for a closed system of just two model economies, which shall interact through a capital transfer. We assume in this sample, that the second economy Y_2 starts 25 years later, and is only half as strong as Y_1. In fig. 11 A we see the not-interacting case: Both economies take the usual course of development of their GDP's Y_1, Y_2. As was to expect, the maximum of GDP Y_2 remains well below the maximum of Y_1.

In the second graph fig.11 B we see the significant effect of a capital transfer in an amount equal to one tenth of the GDP of Y_1 in direction to the smaller economy Y_2 : Now the second economy is boosted dramatically outpacing the first one at its maximum. The GDP Y_1 but stagnates at a pretty good level. Evidently but also is that Y_2 crashes a little bit earlier and far more violent. The priceless favour for economy Y_1 but is, that it will survive over much longer periods than usual. This will happen as long as the stronger economy Y_1, K_1 can internationalize its surplus funds by selling them abroad. By selling debt instruments an economy thus can be maintained very much longer than usual possible.

Globalization is done on trade of real goods and financial products as well. But there is a subtle difference: E.g. the trade of 1 bn USD in cars takes a year of hard work for thousands of people to produce, transport and sell them abroad, the transfer of assets may take just a short click at a computer terminal. The car trade is a balanced act between interchange of money (K) and GDP (Y). Thus trade in real goods will not have such a big influence on the economic evolution as pure financial actions, which is just transferring money between

33 *Dollar, David; Kraay, Aart. "Trade, Growth, and Poverty". Finance and Development. International Monetary Fund. http://www.imf.org/external/pubs/ft/fandd/2001/09/dollar.htm Retrieved 6 June 2011.*

34 In principle our system can be extended to the large scale World-GDP system $\frac{d}{dt}Y_i = \sum b_{0ij} + p_B Y_i - p_n K_i$ with its coupled World-financial system $\frac{d}{dt}K_i = \sum a_{0ij} + p_S Y_i + p_n K_i$ where a_{0ij} and b_{0ij} represent the interactions of the *i-th* with the *j-th* economy.

investors and debtors. This effect[35] was seen statistically e.g. by Dollar and Kraay (IMF).

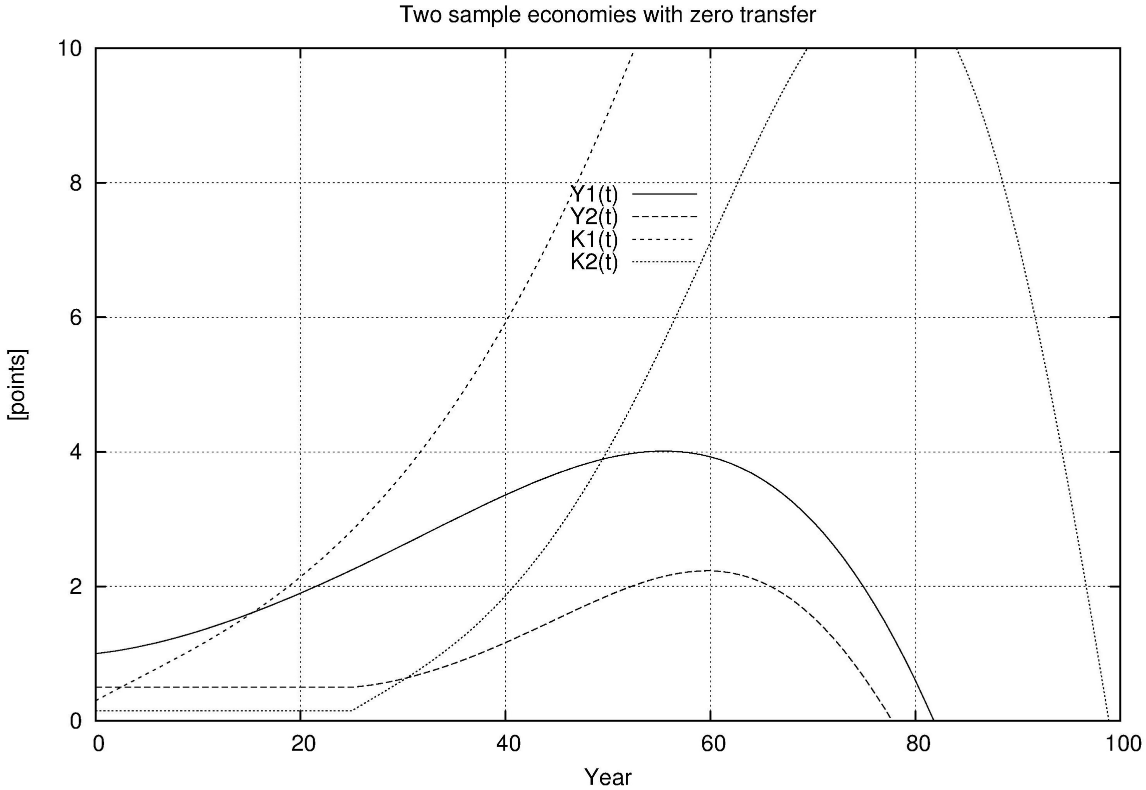

Fig 11 A: Two sample economies without capital transfer in comparison. In the first graph we have two independent economies.

[35] The sample above shows the principle behaviour of strongly interacting economies like the US and China. While the US can perpetuate their heavily indebted economy by selling high amounts of debt instruments, the foreign consumers of those assets may boost their economies "like rockets". But for the price of a coming "sudden death". The historic samples are countless. It hits preferably rising economies, from Argentina over China to Zimbabwe.

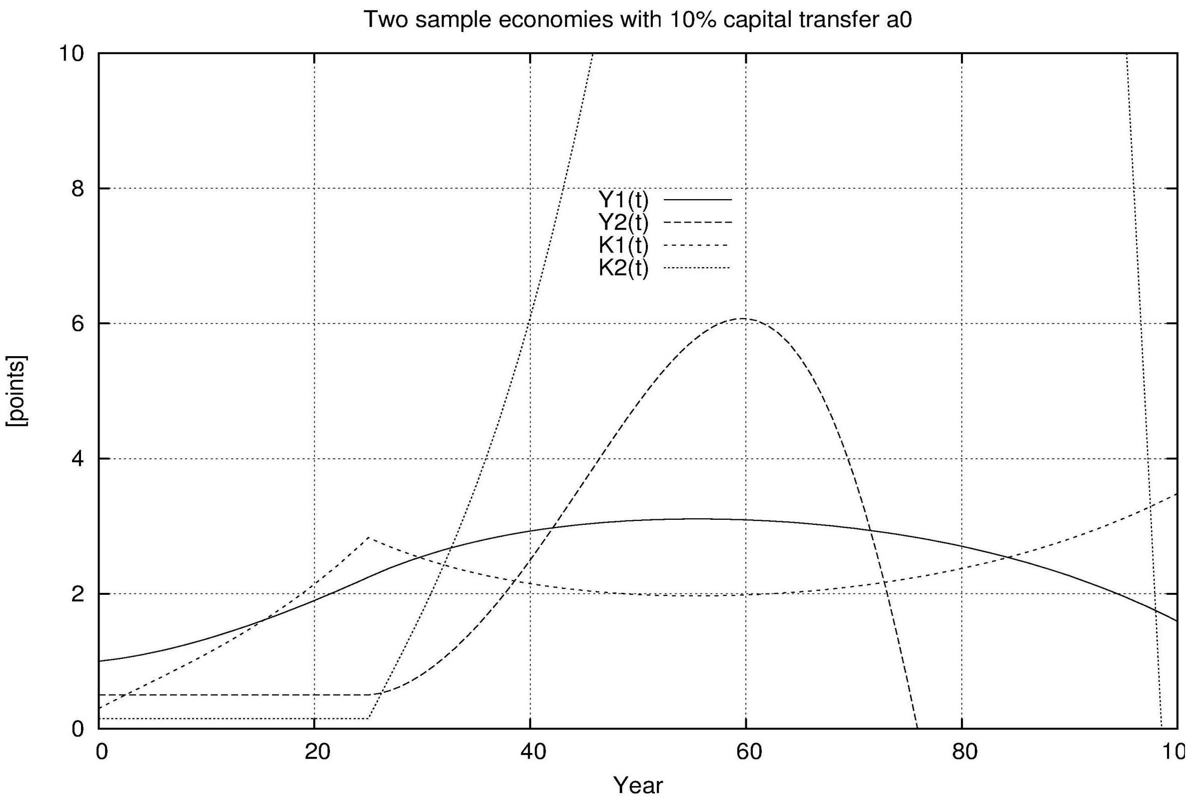

Fig 11 B: The same two sample economies now with capital transfers: Economy 1 sells 10% of its debt instruments to economy 2.

f) Inequality

There are a lot of reasons discussed for the rise and deepening of economical inequality in a society. One reason for sure is the trend to substitute labour with automation by machines and recently by use of computers and software. Which can substitute for a lot of labour, recently including even social work, like e.g. done in call-centres. A lot of labour is also globalized to low-income countries, which means the revenues (part of K) are collected in an economy where the needed work (part of Y) is not balanced for in the GDP.

Another reason is national taxes and fees. We did some maths to this issue using the expanded Fisher equation in chapter 17 of reference [5] "*Rules of Public Debt*". So regularly the majority of these sums are collected on the labour of workers and entrepreneurs alone. E.g. in Germany the states quota of much more than 1000 bn. Euros per year has a share of roughly 15 bn. out of the taxations of wealth, not much more than 1% of the states needs for investments and consumption. This in effect is a steady drain of labour efforts up to the wealthy, thus from GDP Y to capital K. While in early times of a GDP-driven economy this relation will not be of great importance, in the later capital driven phases this feature may dramatically increase financial, economical and finally social crisis of an economy.

Definitely taxation laws are extremely complicated and differ very much from the kind of product, may it be labour, goods or assets. It's neither easy to understand nor in anyway equitable. Taxation employs legions of lawyers and tax consultants and costs a huge amount of money for any actor on the market, firms, workers and even for the tax collectors themselves. Tax inequality by law is a billion dollar market on its own. On other grounds also the amount of taxes and duties seemingly never fit the demand by the states needs. As obviously practically every taxation year the state issues more debt instruments than he needs to refinance his older debt, thus increasing the sum of national debt on a yearly basis.

We may make a provocative revolutionary claim at this point: Is it really necessary that the state has to levy the highly skewed distribution of taxes and duties on such a complicated way? On pure mathematical grounds[36] he could do the issue in a much simpler, and in effect much more equal and fair way.

Basically put, he could completely dispense the collection of taxes, duties and fees. Instead of just issuing his yearly shortfall the state also could print the whole of his needs.

This would not as bad as it sounds. First of all because, as it is also the case with regular taxation, this money is not saved but given in whole to the economy for investments and consumption, thus being one of the main drivers of an economy. This money finally gets its equivalent in GDP goods one to one. Secondly such a tax would benefit, but also hurt, any member of the society in the same equal way. As it is only damage would be to depreciate the monetary value over time. The amount of this depreciation need not to be much more as in the actual system, rather less, as it will be balanced by additional GDP. But this depreciation (which is nothing at all as the usual inflation) will hurt assets as well as labor prices in the same way.

This is in sharp contrast to the actual system, as it hurts labor much more than wealth: Although both are equally affected by the inflation, wages and wins in the real industry are additionally effected by constantly rising taxes and fees. This would come to an end giving reason to more economic equality. The collection of taxes, through government bonds alone had of course to be regulated by strict laws. This means in particular that only the amount will be charged, as it can also cope with the real economy to investment and consumption, including welfare, without overheating the GDP. Such state expenditure must be so scarce that it neither comes directly nor indirectly to greater savings in the economy. Approximately due to excessive public sector wages or to much welfare, which would then flow largely into saving accounts instead into consumption thus hurting growth.

Revolutionary changes to economy in the near future, as some of them were pointed out in this article, won’t be easy to achieve. Exact algebraic analysis of the economy but may lead to deeper insights into otherwise hidden dependencies in an economy, leading to New Economic Thinking at least. Newtonian mechanics of the Economy is at least able to provide food for such thoughts. As like in physics and engineering we may be able to verify such thoughts on ground of the usual experiments-theory-cycles. The price one has to pay for it, is a technical and a political-economic one. Technically its the need to use some classical algebraic techniques as known from physics and engineering also in the Economics. The political-economic one is to accept that the financial part of the economy cannot be taken, not even partly, out of the considerations. The exact opposite is the case - capital drives more than anything else the economy on, both on the good as on the bad side.

[36] Which are the substituting rules in a closed economy by virtue of the expanded Fisher equation. As simply raising taxes and fees through central banks, instead of collecting them in special chosen realms of the economy, in the overall-sum makes no difference. It is just another kind of reallocation between production and wealth.

9. Summary

We proved that classical Newtonian mechanics and physical field theory may be applied successful to macroeconomic systems. The main differences between the Newtonian approach and classical growth models in economy are:

- the whole of all Capital (K, total banks balance sheets) is taken into the calculation
- the constituting equation is a system of two coupled balanced differential equations for Y and K

The first difference is essentially a question of economic thought: it is a farewell to the classical dichotomy claim. It says, that money is not a “God-given” exogenous lubricant but ultimately linked to efforts in the real and yes, also financial, economics. The second difference is of mathematical importance. For a self consistent theory of an economy, the capital market and the real goods markets must also be mathematically coupled in the constituting equations.

We did some sample calculation to show the basic features of the theory. The presented mathematical tools may be extended to handle closed and open economy and their international dependencies as well.

Algebraic modelling has the priceless favour of conserving dependencies through the whole of all calculations. Only this feature gives the possibilities for deep insight into the system under consideration as we know it from theoretical physics. For the price of “tricky” mathematical calculations which sometimes can be done only approximately. On the other side, where analytical integrations are not possible, we can do the job numerically even more exactly with the use of strong computer power and specialized software.

Numerical solutions will be achievable even in large-scale calculations of world’s economies and their interdependencies. Such software in connection with reliable databases of the main economic input parameters could give politics, banks and economics great forecast help for growth prediction and thus in law making and central banks decision.

<u>The Special Need for Statistical Data</u>

The Newtonian model works predictive on its own. We need but reliable data to check if and how incorporated dependencies work and if or not a theoretical input assumption was correct. This is the direct equivalent to experiments in physics and engineering.

This sounds more easier than it is in reality. This is as national statistics provide data based on classical assumptions in macroeconomy, which are based mostly on GDP tasks. Especially in the realm of financial institutions but the data are often incomplete or otherwise not very helpful. So we need all banks assets of any kind, at least their sums. Failing to sum up all assets working in an economy makes prediction weak: Each financial asset is engaged in the interactions of the economy of trade and savings just like any real product. Financial and real products both substitute with any other tradable product.

Also GDP data usually are not provided as raw data in dollar and cents, but in already statistically altered time lines. Besides the question, how exact those numbers really are, there always are some measures on inflation correction applied. But much more worse is “hedonizing” which is altering data in terms of an arbitrary “quality factor”. Naturally this makes GDP data look much better than they really are, but makes them worthless for exact calculations. Institutions should publish raw data at least separately.

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